# The Market Price of Skewness 

JOB MARKET PAPER

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#### Abstract

This paper provides new insights in the skewness risk premium in the stock market. By building strategies which take position in the individual skewness of the constituents of the SP100, we show that the skewness risk premium becomes positive and significant for almost all the stocks after the 2007-2009 financial crisis. We find that this is due to a drastic increase (in absolute value) in the price of the skewness, while we do not find any significant change in the realised skewness of the returns. Consistently, we find that the shape of the average implied volatility smile across stocks becomes steeper after the crisis. Moreover, we find that this pre/post crisis structural change does not apply to the market skewness risk premium, computed as the skew premium of the index SP500.


Keywords : Risk premium, risk neutral skewness, realised skewness, financial crisis, equity
market, empirical asset pricing, trading strategy.

[^0]In this paper we provide strong empirical evidence for the existence of a significant skewness risk premium in the single stock market after the 2007-2009 financial crisis. We build model-free dynamic trading strategies which are bets on the skew and by following the return of these strategies over time we recover the time series of the skew risk premium in the individual stock market. We show that before the crisis the skew risk premium is very heterogeneous among stocks, it often switches sign and it is on average not significant, but after the crisis the skew risk premium becomes significant and positive for almost the totality of the stocks. We find that the price of the skewness increases significantly after the crisis while the realised skewness does not show any significant change. These results are confirmed with a study of the implied volatility function. We find that after the crisis the average slope of the implied volatility smile increases significantly. Interestingly, We find that the skewness risk premium of the SP500 is positive and significant throughout all our sample period and does not experience this structural change.

The risk premium in financial markets arises when the true probability distribution of a financial asset is different from the distribution based on which the price of a contingent claim on the asset is computed, which is called the risk-neutral distribution. The reason for this difference is that investors do not give equal weight to all states of the asset, and, according to their risk-averse preferences, they usually give more weight to the bad states of the asset. Hence, studying the differences between the true probability distribution $\mathbb{P}$ and the risk-neutral probability distribution $\mathbb{Q}$ allow us to study the preferences of investors.

In the equity market, it has been widely documented the existence of a positive first-moment risk premium $E^{\mathbb{P}}\left[r_{t}\right]-E^{\mathbb{Q}}\left[r_{t}\right]=E^{\mathbb{P}}\left[r_{t}\right]-r_{f, t}$, where $r_{t}$ is the asset return and $r_{f, t}$ is risk-free return. This risk premium is modeled in the CAPM framework and in factor models (see e.g. Fama and French (1993), Carhart (1997), Pastor and Stambaugh (2003), Fama and French (2015)).

A recent stream of literature investigates the risk premiums of the higher moments of the return
distribution, in particular the variance risk premium. For example, Bakshi and Kapadia (2003) examine the statistical properties of delta-hedged option portfolios on the SP500 and find that the average gain of the strategy is negative. Similarly, Bollen and Whaley (2004) document the negative returns earned by buyers of out-of-the-money index puts. Carr and Wu (2009) construct strategies which take position in the variance of the asset through the construction of option portfolios, and they find that the average return of the strategy is negative both for the index and for 30 main stocks. Ang et al. (2006) investigate how the stochastic volatility of the market is priced in the cross-section of expected stock returns. They build portfolios of stocks that have different sensitivities to innovations in market volatility and find that stocks with large, positive sensitivities to volatility risk have low average returns. All this evidence is supportive of a negative market volatility risk premium. Investors dislike volatility, because increasing volatility represents a deterioration in investment opportunities. Risk-averse agents demand to hedge against a rise in volatility, thus the $\mathbb{Q}$ price of volatility is higher than the average realised $\mathbb{P}$ volatility, leading to a negative volatility risk premium.

The skewness risk premium has been less studied in the literature, despite its importance. The main reason is that building strategies which are bets on the skewness of the asset is not trivial. Bakshi et al. (2003) develop a methodology to compute the risk-neutral moments of the asset through the construction of option portfolios. They document that the risk-neutral skewness of the SP500 index and of 30 main stocks is negative, and it is in absolute value higher for the index than for the individual stocks. They also show theoretically that, within a power utility economy in which returns are leptokurtic, the risk-neutral implied skew is greater in magnitude than the physical $\mathbb{P}$ skew. Conrad et al. (2013) apply the methodology of Bakshi et al. (2003) to single stocks and find that the more ex-ante negatively skewed returns yield subsequent higher returns. Kozhan et al. (2013) and Schneider and Trojani (2014) develop a methodology for measuring the risk premium in any desired moment of returns. The key feature of the methodology is that it is a trading strategy, so the expected profit from the strategy can
be directly interpreted as a risk premium. They apply the methodology to the SP500 and they find that the index skewness risk premium is positive. Investors like skewness, because positive skewness imply higher probability of having high returns. Hence, risk-averse investors want to hedge against a drop in skewness, thus the $\mathbb{Q}$ price of skewness is lower than the average realised $\mathbb{P}$ skewness. These results are in line with the theoretical model of Bakshi et al. (2003).

In this work, we apply the technology of Schneider and Trojani (2014) to the constituents of the SP100 in order to gain new insights on the characteristics of the skewness risk premium of individual stocks. The trading strategy has the form of a skew swap, where the fixed leg is the $\mathbb{Q}$ skewness computed at the start date of the swap with the price of a complex option portfolio, and the floating leg is the $\mathbb{P}$ skewness, computed as the payoff of the option portfolio plus a continuous delta hedge. The details of the skew swap are given in Section 1. We fix a monthly maturity for the swap and we implement the skew swaps every month throughout our sample period 1996-2015 for each individual stock. The monthly skewness risk premium is then calculated as the payoff of the swap, given by the difference between the floating leg ( $\mathbb{P}$ skewness) and the fixed leg ( $\mathbb{Q}$ skewness). In this way we construct for each stock the time series of its skewness risk premium.

The main result of our study is that there is a structural change in the skewness risk premium after the 2007-2009 financial crisis. Before the crisis, the skew risk premium is positive and significant for only 10 stocks while after the crisis 93 stocks has a positive and significant risk premium. We find that the results are driven by a drastic increase in the price (in absolute value) of the $\mathbb{Q}$ skewness while there is not a significant change in the $\mathbb{P}$ skewness of the stocks. We document that the skewness risk premium of the SP500 is positive and significant throughout all our sample period and does not experience this structural change. Our results are linked with the work of Kelly et al. (2015). In this paper, the authors document that the difference in costs between out-of-the-money options for individual banks and puts on the financial sector index increases after the 2007-2009 crisis. In our work, we find a post-crisis increase in the
individual $\mathbb{Q}$ skewness (measured with a portfolio of out-of-the-money options) for all stocks across different sectors. In addition, we show that, in accordance to our findings, the shape of the average implied volatility function after the crisis steepens, as shown in Figure 4.

The rest of the paper is organised as follows. Section 1 introduces the skew swaps used in our empirical investigation. Section 2 contains the main result of the paper: in Subsection 2.1 we characterize the historical behaviour of the skewness risk premium, Subsection 2.2 documents the post-crisis steepening of the smile and in Subsection 2.3 we show that the $\mathbb{Q}$ skewness is not a forecast of the $\mathbb{P}$ skewness. Finally, Section 3 concludes. In the paper, we use the notation $\mathbb{Q}$ skewness, priced skewness, implied skewness as synonyms for the risk-neutral skewness. Analogously, we use the notation $\mathbb{P}$ skewness for the realised skewness.

## 1 The skewness swap

To investigate the skewness risk premium in the equity market, we need to compare the riskneutral skewness with the skewness of the real distribution of the asset. The difference of the two skewness measures is the risk premium. In addition, we want to study the characteristics of a tradable risk premium, i.e. the return of an investment which is a bet on the skewness.

Recent research proposes to assess ex-ante moments of the equity return distribution based on option prices (see, e.g. Bakshi et al. (2003), Kozhan et al. (2013), Schneider and Trojani (2014)). The common idea behind these studies is that the different option prices across the strikes contain information about the risk neutral distribution of the underlying. By building option portfolios which take long position in out-of-the-money calls and short position in out-of-the-money puts, these studies show how to extrapolate the ex-ante skewness.

Among these studies, the new methodology developed by Schneider and Trojani (2014) stands out because it allows to identify the tradeable risk premiums from the excess return of special swaps. Moreover, their approach allow us to isolate the tradable properties of higher-order risk (for the skewness we are interested in the third moment) from second-order volatility risk.

In their work they apply the methodology to the SP500 index, and we extend their work by applying the methodology to single stocks.

Their approach starts with the definition of the realised divergence between $F_{t_{1}, T}$ and $F_{t_{2}, T}$ associated with a twice-differentiable generating function $\Phi: \mathbb{R} \rightarrow \mathbb{R}$ :

$$
\begin{equation*}
D_{\Phi}\left(F_{t_{2}, T}, F_{t_{1}, T}\right)=\Phi\left(F_{t_{2}, T}\right)-\Phi\left(F_{t_{1}, T}\right)-\Phi^{\prime}\left(F_{t_{1}, T}\right)\left(F_{t_{2}, T}-F_{t_{1}, T}\right) \tag{1}
\end{equation*}
$$

We denote with $F_{t, T}$ the forward price at time $t$ for delivery in $T$. The intuition is that $D_{\Phi}\left(F_{t_{2}, T}, F_{t_{1}, T}\right)$ captures the variation of the forward price between $t_{1}$ and $t_{2}$ measured by the function $\Phi$. For example for the choice $\Phi(x)=\left(x / F_{t_{1}, T}\right)^{2}-1$ we have $D_{\Phi}\left(F_{t_{2}, T}, F_{t_{1}, T}\right)=$ $\left(\frac{F_{t_{2}, T}-F_{t_{1}, T}}{F_{t_{1}, T}}\right)^{2}$ which is a measure of the realised variance. Given a discrete grid of trading dates $0=t_{0}<t_{1}<t_{2}<\ldots<t_{n}=T$, Schneider and Trojani (2014) define the global divergence between $F_{t_{0}, T}$ and $F_{T, T}$ as the sum of the divergences in each period

$$
\begin{equation*}
D I V_{\Phi}\left(F_{0, T}, F_{T, T}\right)=\sum_{i=1}^{n} D_{\Phi}\left(F_{i, T}, F_{i-1, T}\right)=\Phi\left(F_{T, T}\right)-\Phi\left(F_{0, T}\right)-\sum_{i=1}^{n} \Phi^{\prime}\left(F_{i-1, T}\right)\left(F_{i, T}-F_{i-1, T}\right) \tag{2}
\end{equation*}
$$

We use the notation $F_{i, T}:=F_{t_{i}, T}$ for brevity. In order to build a trading strategy whose payoff is the global divergence $D I V_{\Phi}\left(F_{0, T}, F_{T, T}\right)$, we use the following result proved by Carr and Madan (2001):

$$
\begin{equation*}
\Phi(y)-\Phi(x)-\Phi^{\prime}(x)(y-x)=\int_{0}^{x} \Phi^{\prime \prime}(K) P_{T, T}(K) d K+\int_{x}^{\infty} \Phi^{\prime \prime}(K) C_{T, T}(K) d K \tag{3}
\end{equation*}
$$

which holds for every $x$ and $y$ in $\mathbb{R} . P_{T, T}(K)$ is the payoff of the put option with strike $K$ at time $T$ and $C_{T, T}(K)$ is the payoff of the call. By substituting $x=F_{0, T}$ and $y=F_{T, T}$ we obtain
$\Phi\left(F_{T, T}\right)-\Phi\left(F_{0, T}\right)-\Phi^{\prime}\left(F_{0, T}\right)\left(F_{T, T}-F_{0, T}\right)=\int_{0}^{F_{0, T}} \Phi^{\prime \prime}(K) P_{T, T}(K) d K+\int_{F_{0, T}}^{\infty} \Phi^{\prime \prime}(K) C_{T, T}(K) d K$

It is then easy to prove that

$$
\begin{aligned}
D I V_{\Phi}\left(F_{0, T}, F_{T, T}\right)= & \int_{0}^{F_{0, T}} \Phi^{\prime \prime}(K) P_{T, T}(K) d K+\int_{F_{0}, T}^{\infty} \Phi^{\prime \prime}(K) C_{T, T}(K) d K \\
& +\sum_{i=1}^{n-1}\left(\Phi^{\prime}\left(F_{i-1, T}\right)-\Phi^{\prime}\left(F_{i, T}\right)\right)\left(F_{T, T}-F_{i, T}\right)
\end{aligned}
$$

This last equation shows that the realised global divergence $D I V_{\Phi}\left(F_{0, T}, F_{T, T}\right)$ can be replicated with a portfolio of options plus a discrete delta-hedge in the forward market. The price of this strategy is

$$
\begin{aligned}
E_{0}^{\mathbb{Q}}\left[D I V_{\Phi}\left(F_{0, T}, F_{T, T}\right)\right]= & E_{0}^{\mathbb{Q}}\left[\int_{0}^{F_{0, T}} \Phi^{\prime \prime}(K) P_{T, T}(K) d K+\int_{F_{0}, T}^{\infty} \Phi^{\prime \prime}(K) C_{T, T}(K) d K\right] \\
& +E_{0}^{\mathbb{Q}}\left[\sum_{i=1}^{n-1}\left(\Phi^{\prime}\left(F_{i-1, T}\right)-\Phi^{\prime}\left(F_{i, T}\right)\right)\left(F_{T, T}-F_{i, T}\right)\right] \\
& =\frac{1}{B_{0, T}}\left(\int_{0}^{F_{0, T}} \Phi^{\prime \prime}(K) P_{0, T}(K) d K+\int_{F_{0, T}}^{\infty} \Phi^{\prime \prime}(K) C_{0, T}(K) d K\right)
\end{aligned}
$$

where $C_{0, T}(K)$ and $P_{0, T}(K)$ are the prices at time $t_{0}$ of an European call and put with maturity $T$ and strike $K . B_{0, T}$ is the price of a zero-coupon bond with expiration $T$.

The trading strategy which has as payoff the global realised divergence $D I V_{\Phi}\left(F_{0, T}, F_{T, T}\right)$ can therefore be implemented as a swap. The fixed leg of the swap is determined at the start date $t_{0}$ by the option portfolio:

$$
\begin{equation*}
f x l=\frac{1}{B_{0, T}}\left(\int_{0}^{F_{0, T}} \Phi^{\prime \prime}(K) P_{0, T} d K+\int_{F_{0, T}}^{\infty} \Phi^{\prime \prime}(K) C_{0, T} d K\right) \tag{5}
\end{equation*}
$$

The floating leg of the swap is the global realised divergence $D I V_{\Phi}\left(F_{0, T}, F_{T, T}\right)$ which realises its value only at the end date of the swap, which coincides with the maturity $T$ of the options. The value of the floating leg is the sum of the payoff of the option portfolio constructed at the start date of the swap and the payoff of a discrete delta-hedge in the forward market computed
at each time $t_{0}<t_{i}<T$ :

$$
\begin{align*}
f l l & =\left(\int_{0}^{F_{0, T}} \Phi^{\prime \prime}(K) P_{T, T} d K+\int_{F_{0, T}}^{\infty} \Phi^{\prime \prime}(K) C_{T, T} d K\right)  \tag{6}\\
& +\sum_{i=1}^{n-1}\left(\Phi^{\prime}\left(F_{i-1, T}\right)-\Phi^{\prime}\left(F_{i, T}\right)\right)\left(F_{T, T}-F_{i, T}\right) \tag{7}
\end{align*}
$$

Because $E_{0}^{\mathbb{Q}}[f l l]=f x l$, the value of the swap is zero at its inception and all the payments are made at maturity. The floating leg of the swap is the realised divergence ( $\mathbb{P}$ divergence) between $F_{0, T}$ and $F_{T, T}$ associated with the function $\Phi$. The fixed leg of the swap is the ex-ante risk-neutral price of the divergence ( $\mathbb{Q}$ divergence). The gain of the swap strategy is given by the difference between the $\mathbb{P}$ divergence and the $\mathbb{Q}$ divergence and it is a measure of the realised risk premium associated to the divergence generated by the function $\Phi$.

Schneider and Trojani (2014) show that the generating function

$$
\begin{equation*}
\Phi_{2}\left(\frac{x}{F_{0, T}}\right)=-4\left(\left(\frac{x}{F_{0, T}}\right)^{1 / 2}-1\right) \tag{8}
\end{equation*}
$$

generates a swap that well captures the variance of the distribution of the underlying asset. The fixed leg of this swap measures the option implied ex-ante variance and has the following expression:

$$
\begin{equation*}
V A R_{t, T}^{\mathbb{Q}}=\frac{2}{B_{t, T}}\left(\int_{0}^{F_{t, T}} \frac{\sqrt{\frac{K}{F_{t, T}}} P_{t, T}(K)}{K^{2}} d K+\int_{F_{t, T}}^{\infty} \frac{\sqrt{\frac{K}{F_{t, T}}} C_{t, T}(K)}{K^{2}} d K\right) \tag{9}
\end{equation*}
$$

The generating function

$$
\begin{equation*}
\Phi_{3}\left(\frac{x}{F_{0, T}}\right)=-4\left(\frac{x}{F_{0, T}}\right)^{1 / 2} \log \left(\frac{x}{F_{0, T}}\right) \tag{10}
\end{equation*}
$$

generates a swap that captures the skewness of the distribution of the underlying asset. Schneider and Trojani (2014) deduce that the fixed leg of the swap generated by $\Phi_{3}$ captures the
third forward-neutral moment of the $\log$ returns $\log \left(F_{T, T} / F_{0, T}\right)$ while being independent of the moments less than 3. In this case, the fixed leg of the swap is a measure of the risk neutral skewness of the asset and has the following expression:
$S K E W_{t, T}^{\mathbb{Q}}=\frac{1}{B_{t, T}}\left(\int_{F_{t, T}}^{\infty} \log \left(\frac{K}{F_{t, T}}\right) \frac{\sqrt{\frac{K}{F_{t, T}}} C_{t, T}(K)}{K^{2}} d K-\int_{0}^{F_{t, T}} \log \left(\frac{F_{t, T}}{K}\right) \frac{\sqrt{\frac{K}{F_{t, T}}} P_{t, T}(K)}{K^{2}} d K\right)$

The floating leg of the swap is the realization of the conditional skewness under the true probability measure $\mathbb{P}$. The realised gain of a swap holder who pays fixed and receive floating is calculated at maturity $T$ as the difference between the floating leg and the fixed leg of the swap. This difference represents the realization of the skewness risk premium. Throughout our empirical study we apply the skew swap of Schneider and Trojani (2014) with $\Phi=\Phi_{3}$ to 100 stocks to obtain the time series of the skewness risk premium for each stock.

## 2 Data and empirical methodology

We apply the skewness swaps introduced in Section 1 to all the components of the SP100 separately. The list of the actual components is taken from Compustat database as of March 2016. We then use all the available data coverage of options of the Optionmetrics database which starts in January 1996 and ends in August 2015. The data on the security price, the dividend distribution history and as well the interest rates is taken from Optionmetrics. We fix a monthly horizon for the skewness swaps, starting and ending on the third Friday of each month, consistently with the maturity structure of option data.

Because the stock options are American, we cannot directly apply the methodology described in Section 1 which is based on portfolios of European options. In order to overcome this issue, we consider only the periods in which the stock doesn't distribute dividends. During this periods, the price of the American calls are equal to the price of European calls and we replicate the
position in the European puts via the put-call parity:

$$
\begin{equation*}
P_{0, T}(K)=C_{0, T}(K)-S_{0}+K B_{0, T} \tag{12}
\end{equation*}
$$

where $C_{0, T}(K)$ and $P_{0, T}(K)$ are the prices at time $t_{0}$ of an European call and put with maturity $T$ and strike $K, B_{0, T}$ is the price of a zero-coupon bond with expiration $T$ and $S_{0}$ is the current stock price at time $t_{0}$. After this period selection, we have on average 150 strategies for each stock covering the full data sample period. The fixed leg of the swap is computed at the start date of the swap by building the portfolio of options described in equation (11). Equation (11) is written for a complete option market in which a continuum of options is available covering all the strikes in the range $[-\infty,+\infty]$. In practice we have only a finite number of strikes for each date. We thus implement a discrete approximation of equation (11). Suppose that at time $t_{0}$, the start date of our swap, there are $N$ calls and $N$ puts traded in the market. We order the strikes of the calls such that $K_{1}<\ldots<K_{M c} \leq F_{0, T}<K_{M c+1}<\ldots<K_{N}$ and the strikes of the puts such that $K_{1}<\ldots<K_{M p} \leq F_{0, T}<K_{M p+1}<\ldots<K_{N} . F_{0, T}$ is the forward price of the stock at time $t_{0}$ for delivery in $T$ and it is calculated as $S_{0} e^{r T}$ where $r$ is the one-month risk free rate calculated as the interest rate of a zero-coupon bond with one month maturity.

We approximate $S K E W_{0, T}^{\mathbb{Q}}$ with the following quadrature formula:
$\widehat{S K E W_{0, T}^{\mathbb{Q}}}=\frac{1}{B_{0, T}}\left(\sum_{i=M c+1}^{N} \log \left(\frac{K_{i}}{F_{0, T}}\right) \frac{\sqrt{\frac{K_{i}}{F_{0, T}}} C_{0, T}\left(K_{i}\right)}{K_{i}^{2}} \Delta K_{i}-\sum_{i=1}^{M p} \log \left(\frac{F_{0, T}}{K_{i}}\right) \frac{\sqrt{\frac{K_{i}}{F_{0, T}}} P_{0, T}\left(K_{i}\right)}{K_{i}^{2}} \Delta K_{i}\right)$
where

$$
\Delta K_{i}= \begin{cases}\left(K_{i+1}-K_{i-1}\right) / 2 & \text { if } 1<i<N \\ \left(K_{2}-K_{1}\right) & \text { if } i=1 \\ \left(K_{N}-K_{N-1}\right) & \text { if } i=N\end{cases}
$$

The usual option data filtering is applied: we exclude options with negative bid-ask spreads,
with an implied volatility smaller than 0.001 or greater than 9 , with a Gamma less than zero and with a Delta bigger than 0.98 or smaller than 0.02 .

The floating leg is composed by two parts: the payoff of the option portfolio (13) at maturity $T$ plus the delta hedge given by equation (7). We implement the delta-hedge each day $t_{i}$, starting from day $t_{1}$ (the day after the start date of the swap) until day $t_{n-1}$ (the day before the maturity of the swap) by buying $\left(\Phi^{\prime}\left(F_{i-1, T}\right)-\Phi^{\prime}\left(F_{i, T}\right)\right)$ forwards on $S_{T}$. The payoff of each daily hedge is $\left(\Phi^{\prime}\left(F_{i-1, T}\right)-\Phi^{\prime}\left(F_{i, T}\right)\right)\left(F_{T, T}-F_{i, T}\right)$ and it is realised at the end date of the swap. All the payments are done at the maturity of the options, which is fixed as the end date (settlement date) of the swap. The realised risk premium of each strategy is calculated at maturity as the difference between the floating leg ( $\mathbb{P}$ skewness) and the fixed leg of the swap ( $\mathbb{Q}$ skewness). We then standardize this difference by variance in order to have a scale-invariant skewness risk premium $R P$ which is comparable across stocks:

$$
R P=\frac{f l l-f x l}{\left(V A R_{0, T}^{\mathbb{Q}}\right)^{3 / 2}}
$$

where $V A R_{0, T}^{\mathbb{Q}}$ is defined in equation (9) and it is calculated using the same numerical approximation used to calculate $S \widetilde{K E W_{0, T}^{\mathbb{Q}}}$ in equation (13).

Table 1 presents a general description of the securities analysed, their data availability, the number of skew swaps implemented, and the average number of options used to calculate the $\mathbb{Q}$ skewness. We see that we have a good data coverage.

### 2.1 Historical behaviour of the skewness risk premium

We implement the monthly skew swap strategy independently for each stock. Thus, each stock of our sample will have a time series of realised skewness risk premium. In Figure 1 we plot the average risk premium together with the $5 \%$ and $95 \%$ quantiles. We notice that there is a high heterogeneity among stocks, especially in the first part of our sample (1996-2000). The
risk premium takes positive and negative values with a high dispersion among stocks. Because the risk premium is calculated as the difference between the realised skewness and the priced skewness, a negative peak of the risk premium happens when the realised skewness has an unexpected decline, which was not priced in the ex-ante skewness. We can easily connect most of the negative peaks with the main recent crisis: the financial crisis of 2007-2009, the Gulf War II of 2003 and the Asian and Russian financial crisis of 1997 and 1998 respectively. We see that during the financial crisis of 2007-2009 the skewness risk premium reaches his lowest level of our sample, and after the crisis there is an upward shift of the range of the risk premiums. Table 2 reports the average risk premium across the stocks before and after the financial crisis. The results are very strong: before the financial crisis the average risk premium is only -0.0518 and moreover it is significant only for 10 stocks. After the financial crisis, the average risk premium becomes 1.282 and it is significant for 93 stocks. Table 3 reports the individual average risk premium for each stock together with the t-statistics. A positive risk premium implies that the priced skewness is less than the realised skewness, but because the priced skewness is generally negative, a positive risk premium implies that the priced skewness is more negative than the realised skewness. An investor who buys skewness will on average make profit, while bearing the risk of a sudden decrease in the realised skewness, i.e. a crash of the asset. Kozhan et al. (2013) and Schneider and Trojani (2014) already documented the existence of a positive skewness risk premium in the equity index market. Our work extend their results by showing that also in the single stock market there is a significant positive risk premium, but only after the 2007-2009 financial crisis.

In Figure 2 and 3 we plot the time series of the priced skewness ( $\mathbb{Q}$ skewness) and the realised skewness ( $\mathbb{P}$ skewness) respectively. We can see from Figure 2 that the priced skewness is on average always negative with a drop in the level after the 2007-2009 financial crisis. In addition, while before the crisis there is a high dispersion in the sign of the priced skewness, after the crisis the skewness becomes negative for almost the totality of the stocks. The time series of
the realised skewness presented in Figure 3 does not show any post-crisis pattern, except that the skewness heterogeneity among stocks diminishes after the crisis. In Table 2 we report the average value of the priced skewness and realised skewness before and after the crisis. We can see that while the priced skewness decreases from -0.3373 to -1.3579 the realised skewness increases from -0.3891 to -0.0759 . To test the significance in the change of the average priced and realised skewness, we compute for each stock the two-sample t-test for equal means. In detail, we divide the time series of the priced skewness and realised skewness of each stock in two samples (pre and post crisis) and we test if the two sample means are equal. The results are reported in Table 3. 84 stocks experience a significant decrease in the priced skewness after the crisis while only 4 stocks show a significant change in the realised skewness. These results document that the significance of the skewness risk premium after the crisis is not due to a change in the real distribution of the underlying stock, but it is due to a drastic change in the priced skewness of the stocks.

We test the difference in the $\mathbb{P}$ skewness of the underlying stock distribution also from un unconditional point of view. We take the pre-crisis time series of the daily returns for each stock and we compute the empirical skewness. Then, we build a confidence interval for the empirical skewness with a bootstrap procedure with 2000 resampling. We finally compute the empirical skewness of the returns in the post-crisis sample and we check if it is inside or outside the confidence interval. We find that 19 stocks have a statistically significant decrease in the unconditional skewness and 12 stocks have a statistically significant increase in the unconditional skewness. We conclude that there is not a strong homogeneous change of the empirical skewness before and after the crisis. We compute the same exercise using the coefficient of asymmetry used by Conrad et al. (2013) and we obtain similar results.

As a robustness check, we compute the time series of the price $\mathbb{Q}$ skewness with the methodology of Bakshi et al. (2003). They construct a measure of risk-neutral skewness through the price of a cubic contract which can be replicated with a portfolio of options. We calculate the
one month $\mathbb{Q}$ skewness with the methodology of Bakshi et al. (2003) for each stock at each start date of our swap contracts. We thus have one time series of $\mathbb{Q}$ skewness for each stock. As before, we divide each time series in two subsamples, pre and post crisis, and we test if the two sample means are equal. The results are reported in Table 5 . We find that 82 out of 100 stocks have a significant decrease in the priced $\mathbb{Q}$ skewness after the financial crisis, thus confirming our previous results.

We apply the skew swap strategy to the time series of the SP500 in order to compare the results obtained for the individual stocks with the market. The results are presented in the first line of Table 3. We find that, in accordance with other studies (see e.g. Bakshi et al. (2003), Kozhan et al. (2013), Schneider and Trojani (2014)), the market skewness risk premium is positive and significant throughout the entire sample period and it is two/three times higher than the risk premium of individual stocks (4.062 against 1.2820 ). The market priced $\mathbb{Q}$ skewness is more negative than the priced $\mathbb{Q}$ skewness of the individual stock ( -4.889 against -1.3579 ). Interestingly, the SP500 doesn't experience a significant change in the $\mathbb{Q}$ price of the skewness before and after the financial crisis. The t-statistics show that there is not a significant change neither in the average market $\mathbb{Q}$ skewness nor in the realised market $\mathbb{P}$ skewness. The structural change that we find in the skewness risk premium of the stock does not apply to the SP500. Our results are connected to the work of Kelly et al. (2015), who find that the difference in costs between out-of-the-money options for individual banks and puts on the financial sector index increases after the 2007-2009 crisis. We find a complementary result: while the out-of-the-money puts on single stocks become more expensive after the financial crisis, the options on the index do not experience the same change.

Table 2 and 3 also report the results on the risk premium, the priced $\mathbb{Q}$ skewness and the realised $\mathbb{P}$ skewness during the financial crisis. We find that during the crisis the average risk premium is negative (due to the negative peaks of the $\mathbb{P}$ skewness) but it is significant only for 2 stocks. The $\mathbb{Q}$ skewness is negative and slightly lower than the $\mathbb{Q}$ skewness before the
crisis. However these results have to be taken with caution, because during the short-sale ban of 2008 and the restrictions on short-sale during the crisis our skew swap could not have been implemented.

### 2.2 The pre/post crisis implied volatility smile

In the previous section we show that the skewness risk premium becomes positive and significant for the quasi-totality of the stocks after the financial crisis of 2007-2009. We then show that this change is due to a significant decrease in the fixed leg of the swap, which represents the priced $\mathbb{Q}$ skewness, while we don't find a significant change in the real $\mathbb{P}$ skewness of the assets distribution. Based on this result, the slope of the implied volatility smile of the stocks, which represents the $\mathbb{Q}$ skewness, has to be in absolute value higher after the crisis. In other words, we should find that the implied volatility smile steepens after the crisis.

To test this hypothesis, we build an average implied volatility smile across the stocks before and after the crisis. First, we divide our sample period in two subsamples: the pre-crisis sample (1996-August 2007) and the post-crisis sample (June 2009-August 2015). Then for each stock we compute the average daily implied volatility smile and we average the results in the pre-crisis sample and in the post-crisis sample. Finally we average the results across the stocks in each of the two samples. In order to build the daily average implied volatility smile for each stock we follow Bollen and Whaley (2004) and we divide all the options available (both calls and puts) with maturity up to one year in five moneyness categories according to their deltas. We then average the implied volatility of the options in each category, where we use the implied volatility provided by Optionmetrics, which takes into account the early exercise of the options. The results are displayed in Figure 4. We first note a decrease in the level of the implied volatility smile after the crisis. In order to better visualize the different slope of the smiles we overlap the two curves by shifting up the post-crisis smile, so that the two curves have the same at-the-money volatility. In accordance to our study, we see a strong steepening of the implied
volatility smile after the crisis. The out-of-the-money puts and in-the-money calls have become $12.5 \%$ more expensive while the out-of-the-money calls and the in-the-money puts have become slightly cheaper. In the same way in which Rubinstein (1994) and Jackwerth and Rubinstein (1996) found a change in the shape of the implied volatility smile after the 1987 crash, we find a steepening of the smile after the financial crisis of 2007. This additional piece of evidence proves that the results of Section 2.1 are not driven by the new methodology we employ, and as well they are not driven by the tenor of the strategies (1 month) nor by the fact that we implement the skew swaps only in periods without dividend distributions.

### 2.3 Predictive regressions

We test whether the ex-ante $\mathbb{Q}$ skewness is a predictor of the subsequent realised $\mathbb{P}$ skewness. We run for each stock in our sample the following standard expectations hypothesis regression:

$$
f l l_{i, t}=\alpha_{0}+\alpha_{1} f x l_{i, t}+\epsilon_{t}
$$

where $f l l_{i, t}$ is the the floating leg ( $\mathbb{P}$ skewness) of the skew swap of the month $t$ for the stock $i$ and $f x l_{i, t}$ is the fixed leg $(\mathbb{Q}$ skewness) of the same skew swap. This is a predictive regression because the two legs are not contemporaneous: the fixed leg $f x l_{i, t}$ is determined at the start date of the swap, while the floating leg $f l l_{i, t}$ is determined only at the end date of the swap.

Table 6 reports the average values of $\alpha_{0}$ and $\alpha_{1}$ among stocks together with the number of stocks for which $\alpha_{0}$ is significantly different than zero $\left(N_{\alpha_{0}}\right)$ and the number of stocks for which $\alpha_{1}$ is significantly different than zero $\left(N_{\alpha_{1}}\right) . \overline{R^{2}}$ is the average $R^{2}$ of the regressions. We see that the predictive power of the fixed leg on the floating leg is very low. Only 5 stocks have a positive and significant $\alpha_{1}$ and the average $R^{2}$ is less than $1 \%$. When we run the regression separately in the pre/post crisis subsamples we see that before the crisis the predictive power of the fixed leg was a bit higher. Indeed, in the pre-crisis regressions 24 stocks show a significant positive
value for $\alpha_{1}$. After the crisis all the predictability disappears, as if the two legs of the skew swap were determined by different factors. This result might be due to the segmentation between the option market and the stock market: the option traders are different investors than the stock traders, thus the determinants of the $\mathbb{Q}$ skewness might be different than the determinants of the $\mathbb{P}$ skewness.

## 3 Conclusions

In this work we implement a trading strategy for investigating the risk premium associated with the third moment of the return distribution. The strategy involves buying and selling out-of-the-money puts and call options in order to take position in the underlying skewness and subsequently hedge in the forward market. In this way we obtain a strategy which is independent from the first and second moment of the underlying and it is a pure bet on the skewness. The return of the strategy measures the skewness risk premium. We apply this strategy to the 100 constituents of the SP100 in the period 1996-2015. We find that after the financial crisis of 20072009 the skewness risk premium is positive and significant for almost all stocks, while before the crisis the results are not significant. A positive skewness risk premium implies that the price of the skewness $(\mathbb{Q}$ skewness) is lower than the realised skewness ( $\mathbb{P}$ skewness). These results are consistent with the theoretical model of Bakshi et al. (2003), which shows that because investors preferences are towards a positive skewness the price of the skewness should be lower than the realised skewness. The market skewness risk premium, measured as the skewness risk premium of the SP500, does not show this pre/post crisis structural change. It is positive and significant throughout the full sample 1996-2015.

The next step is to study what are the economic drivers of the skewness risk premium and what is the connection between the market skewness risk premiums and the risk premiums on individual assets. Given that the correlation in the equity market increased after the financial crisis, it will be interesting to study how much of the skewness risk premium of the single stocks
is due to the covariation of the asset with the market and how much is left as an idiosyncratic component.

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premiums of all the stocks and we plot the $5 \%$ quantile, $50 \%$ quantile and $95 \%$ quantile.
Fixed leg of the skew swap ( $\mathbb{Q}$ skewness)


| 1998 | 2000 | 2002 | 2004 | 2006 | 2008 | 2010 | 2012 | 2014 | 2016 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Floating leg of the skew swap ( $\mathbb{P}$ skewness)


| 1998 | 2000 | 2002 | 2004 | 2006 | 2008 | 2010 | 2012 | 2014 | 2016 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Figure 3: The figure shows the times series of the floating leg ( $\mathbb{P}$ skewness) of the skewness swap for all the
time, we sort the floating legs of all the stocks and we plot the $5 \%$ quantile, $50 \%$ quantile and $95 \%$ quantile.

The implied volatility smile pre/post crisis


Figure 4: The figure shows the implied volatility smile before and after the financial crisis of 2007. The implied volatility function is the average implied volatility of options in five moneyness categories based on option delta, as described in Table 4. Implied volatilities are computed daily for each stock separately and then averaged across stocks. We plot the two curves in the same graph under different scale in order to overlap the two smiles at their at-the-money volatility to better visualize their different slope. The left $y$-axis scale is simply a shift of the right $y$-axis scale.

## Descriptive table of the securities

|  | Ticker | Full name | Start date | End date | N swaps | N options |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | SPX | S\&P 500 Index | 01-Jan-1996 | 31-Aug-2015 | 235 | 47.64 |
| 1 | AAPL | 'APPLE INC' | 15-Mar-96 | 17-Jul-15 | 218 | 24.52 |
| 2 | ABBV | 'ABBVIE INC' | 15-Feb-13 | 21-Aug-15 | 21 | 18.95 |
| 3 | ABT | 'ABBOTT LABORATORIES' | 16-Feb-96 | 21-Aug-15 | 155 | 7.93 |
| 4 | ACN | 'ACCENTURE PLC' | 16-Nov-01 | 21-Aug-15 | 150 | 7.79 |
| 5 | AGN | 'ALLERGAN PLC' | 16-Feb-96 | 21-Aug-15 | 234 | 8.58 |
| 6 | AIG | 'AMERICAN INTERNATIONAL GROUP' | 16-Feb-96 | 21-Aug-15 | 169 | 11.92 |
| 7 | ALL | 'ALLSTATE CORP' | 16-Feb-96 | 21-Aug-15 | 154 | 7.94 |
| 8 | AMGN | 'AMGEN INC' | 16-Feb-96 | 17-Jul-15 | 54 | 14.83 |
| 9 | AMZN | 'AMAZON.COM INC' | 19-Dec-97 | 21-Aug-15 | 210 | 22.12 |
| 10 | AXP | 'AMERICAN EXPRESS CO' | 16-Feb-96 | 21-Aug-15 | 157 | 9.96 |
| 11 | BA | 'BOEING CO' | 15-Mar-96 | 17-Jul-15 | 156 | 10.11 |
| 12 | BAC | 'BANK OF AMERICA CORP' | 16-Feb-96 | 21-Aug-15 | 157 | 6.58 |
| 13 | BIIB | 'BIOGEN INC' | 16-Feb-96 | 21-Aug-15 | 233 | 15.66 |
| 14 | BK | 'BANK OF NEW YORK MELLON CORP' | 15-Mar-96 | 17-Jul-15 | 145 | 8.00 |
| 15 | BLK | 'BLACKROCK INC' | 21-Apr-06 | 21-Aug-15 | 76 | 18.79 |
| 16 | BMY | 'BRISTOL-MYERS SQUIBB CO' | 16-Feb-96 | 21-Aug-15 | 154 | 8.25 |
| 17 | BRK | 'BERKSHIRE HATHAWAY' | 21-Mar-97 | 21-Aug-15 | 74 | 15.61 |
| 18 | C | 'CITIGROUP INC' | 15-Mar-96 | 17-Jul-15 | 151 | 9.72 |
| 19 | CAT | 'CATERPILLAR INC' | 16-Feb-96 | 21-Aug-15 | 155 | 10.02 |
| 20 | CELG | 'CELGENE CORP' | 15-Mar-96 | 21-Aug-15 | 230 | 9.98 |
| 21 | CL | 'COLGATE-PALMOLIVE CO' | 15-Mar-96 | 17-Jul-15 | 144 | 7.70 |
| 22 | CMCSA | 'COMCAST CORP' | 16-Feb-96 | 21-Aug-15 | 190 | 6.23 |
| 23 | COF | 'CAPITAL ONE FINANCIAL CORP' | 15-Mar-96 | 17-Jul-15 | 155 | 11.17 |
| 24 | COP | 'CONOCOPHILLIPS' | 15-Mar-96 | 17-Jul-15 | 155 | 8.94 |
| 25 | COST | 'COSTCO WHOLESALE CORP' | 16-Feb-96 | 17-Jul-15 | 187 | 9.21 |
| 26 | CSCO | 'CISCO SYSTEMS INC' | 16-Feb-96 | 21-Aug-15 | 212 | 6.42 |
| 27 | CVS | 'CVS HEALTH CORP' | 16-Feb-96 | 17-Jul-15 | 146 | 8.12 |
| 28 | CVX | 'CHEVRON CORP' | 15-Mar-96 | 17-Jul-15 | 153 | 9.72 |
| 29 | DD | 'DU PONT (EI) DE NEMOURS' | 15-Mar-96 | 19-Jun-15 | 155 | 9.36 |
| 30 | DHR | 'DANAHER CORP' | 16-Feb-96 | 21-Aug-15 | 154 | 6.96 |
| 31 | DIS | 'DISNEY (WALT) CO' | 16-Feb-96 | 21-Aug-15 | 205 | 8.47 |
| 32 | DOW | 'DOW CHEMICAL' | 16-Feb-96 | 21-Aug-15 | 156 | 8.77 |
| 33 | DUK | 'DUKE ENERGY CORP' | 15-Mar-96 | 17-Jul-15 | 154 | 4.53 |
| 34 | EMC | 'EMC CORP/MA' | 16-Feb-96 | 21-Aug-15 | 222 | 6.52 |
| 35 | EMR | 'EMERSON ELECTRIC CO' | 16-Feb-96 | 17-Jul-15 | 155 | 7.46 |
| 36 | EXC | 'EXELON CORP' | 15-Mar-96 | 17-Jul-15 | 153 | 7.03 |
| 37 | F | 'FORD MOTOR CO' | 15-Mar-96 | 17-Jul-15 | 172 | 3.35 |
| 38 | FB | 'FACEBOOK INC' | 15-Jun-12 | 21-Aug-15 | 39 | 27.70 |
| 39 | FDX | 'FEDEX CORP' | 16-Feb-96 | 21-Aug-15 | 178 | 9.14 |
| 40 | FOXA | 'TWENTY-FIRST CENTURY FOX INC' | 21-Mar-97 | 21-Aug-15 | 183 | 4.93 |
| 41 | GD | 'GENERAL DYNAMICS CORP' | 16-Feb-96 | 21-Aug-15 | 157 | 8.03 |
| 42 | GE | 'GENERAL ELECTRIC CO' | 16-Feb-96 | 21-Aug-15 | 155 | 6.70 |
| 43 | GILD | 'GILEAD SCIENCES INC' | 16-Feb-96 | 21-Aug-15 | 229 | 11.56 |
| 44 | GM | 'GENERAL MOTORS CO' | 17-Dec-10 | 21-Aug-15 | 51 | 14.78 |
| 45 | GOOGL | 'ALPHABET INC' | 17-Sep-04 | 21-Aug-15 | 131 | 65.60 |
| 46 | GS | 'GOLDMAN SACHS GROUP INC' | 17-Sep-99 | 21-Aug-15 | 129 | 17.64 |


| 47 | HAL | 'HALLIBURTON CO' | 19-Apr-96 | 21-Aug-15 | 154 | 10.04 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 48 | HD | 'HOME DEPOT INC' | 16-Feb-96 | 21-Aug-15 | 154 | 9.02 |
| 49 | HON | 'HONEYWELL INTERNATIONAL INC' | 15-Mar-96 | 17-Jul-15 | 154 | 8.24 |
| 50 | IBM | 'INTL BUSINESS MACHINES CORP' | 15-Mar-96 | 17-Jul-15 | 156 | 16.22 |
| 51 | INTC | 'INTEL CORP' | 15-Mar-96 | 17-Jul-15 | 154 | 7.67 |
| 52 | JNJ | 'JOHNSON \& JOHNSON' | 15-Mar-96 | 17-Jul-15 | 155 | 8.66 |
| 53 | JPM | 'JPMORGAN CHASE \& CO' | 16-Feb-96 | 21-Aug-15 | 155 | 10.59 |
| 54 | KMI | 'KINDER MORGAN INC' | 18-Mar-11 | 17-Jul-15 | 36 | 9.76 |
| 55 | KO | 'COCA-COLA CO' | 16-Feb-96 | 21-Aug-15 | 155 | 7.91 |
| 56 | LLY | 'LILLY (ELI) \& CO' | 15-Mar-96 | 17-Jul-15 | 155 | 9.71 |
| 57 | LMT | 'LOCKHEED MARTIN CORP' | 16-Feb-96 | 21-Aug-15 | 156 | 7.64 |
| 58 | LOW | 'LOWE'S COMPANIES INC' | 16-Feb-96 | 17-Apr-15 | 130 | 6.23 |
| 59 | MA | 'MASTERCARD INC' | 16-Jun-06 | 21-Aug-15 | 75 | 37.49 |
| 60 | MCD | 'MCDONALD'S CORP' | 16-Feb-96 | 21-Aug-15 | 181 | 7.69 |
| 61 | MDLZ | 'MONDELEZ INTERNATIONAL INC' | 20-Jul-01 | 21-Aug-15 | 110 | 7.14 |
| 62 | MDT | 'MEDTRONIC PLC' | 16-Feb-96 | 21-Aug-15 | 156 | 9.85 |
| 63 | MET | 'METLIFE INC' | 18-Aug-00 | 17-Jul-15 | 157 | 9.88 |
| 64 | MMM | '3M CO' | 16-Feb-96 | 17-Jul-15 | 154 | 10.04 |
| 65 | MO | 'ALTRIA GROUP INC' | 16-Feb-96 | 21-Aug-15 | 152 | 8.14 |
| 66 | MON | 'MONSANTO CO' | 17-Nov-00 | 21-Aug-15 | 118 | 10.64 |
| 67 | MRK | 'MERCK \& CO' | 16-Feb-96 | 21-Aug-15 | 156 | 10.21 |
| 68 | MS | 'MORGAN STANLEY' | 15-Mar-96 | 17-Jul-15 | 154 | 8.97 |
| 69 | MSFT | 'MICROSOFT CORP' | 16-Feb-96 | 17-Jul-15 | 172 | 9.74 |
| 70 | NEE | 'NEXTERA ENERGY INC' | 16-Feb-96 | 21-Aug-15 | 157 | 5.12 |
| 71 | NKE | 'NIKE INC' | 16-Feb-96 | 21-Aug-15 | 156 | 10.28 |
| 72 | ORCL | 'ORACLE CORP' | 16-Feb-96 | 21-Aug-15 | 188 | 7.04 |
| 73 | OXY | 'OCCIDENTAL PETROLEUM CORP' | 16-Feb-96 | 21-Aug-15 | 156 | 8.84 |
| 74 | PCLN | 'PRICELINE GROUP INC' | 20-Aug-99 | 21-Aug-15 | 186 | 46.91 |
| 75 | PEP | 'PEPSICO INC' | 16-Feb-96 | 21-Aug-15 | 156 | 8.78 |
| 76 | PFE | 'PFIZER INC' | 15-Mar-96 | 17-Jul-15 | 153 | 6.42 |
| 77 | PG | 'PROCTER \& GAMBLE CO' | 16-Feb-96 | 17-Jul-15 | 154 | 8.61 |
| 78 | PM | 'PHILIP MORRIS INTERNATIONAL' | 18-Apr-08 | 21-Aug-15 | 50 | 13.27 |
| 79 | PYPL | 'PAYPAL HOLDINGS INC' | - | - | - | - |
| 80 | QCOM | 'QUALCOMM INC' | 16-Feb-96 | 21-Aug-15 | 175 | 11.37 |
| 81 | RTN | 'RAYTHEON CO' | 16-Feb-96 | 21-Aug-15 | 159 | 7.09 |
| 82 | SBUX | 'STARBUCKS CORP' | 16-Feb-96 | 17-Jul-15 | 133 | 8.55 |
| 83 | SLB | 'SCHLUMBERGER LTD' | 16-Feb-96 | 21-Aug-15 | 152 | 11.17 |
| 84 | SO | 'SOUTHERN CO' | 15-Mar-96 | 17-Jul-15 | 155 | 5.05 |
| 85 | SPG | 'SIMON PROPERTY GROUP INC' | 16-Feb-96 | 17-Jul-15 | 145 | 7.95 |
| 86 | T | 'AT\&T INC' | 16-Feb-96 | 21-Aug-15 | 156 | 6.02 |
| 87 | TGT | 'TARGET CORP' | 15-Mar-96 | 15-May-15 | 138 | 9.13 |
| 88 | TWX | 'TIME WARNER INC' | 16-Feb-96 | 21-Aug-15 | 188 | 10.89 |
| 89 | TXN | 'TEXAS INSTRUMENTS INC' | 16-Feb-96 | 17-Jul-15 | 156 | 9.04 |
| 90 | UNH | 'UNITEDHEALTH GROUP INC' | 16-Feb-96 | 21-Aug-15 | 196 | 9.69 |
| 91 | UNP | 'UNION PACIFIC CORP' | 16-Feb-96 | 21-Aug-15 | 156 | 10.17 |
| 92 | UPS | 'UNITED PARCEL SERVICE INC' | 21-Apr-00 | 17-Jul-15 | 123 | 9.41 |
| 93 | USB | 'U S BANCORP' | 16-Feb-96 | 21-Aug-15 | 157 | 7.02 |
| 94 | UTX | 'UNITED TECHNOLOGIES CORP' | 16-Feb-96 | 17-Jul-15 | 156 | 8.84 |
| 95 | V | 'VISA INC' | 18-Apr-08 | 17-Jul-15 | 59 | 21.13 |
| 96 | VZ | 'VERIZON COMMUNICATIONS INC' | 16-Feb-96 | 21-Aug-15 | 156 | 7.42 |
| 97 | WBA | 'WALGREENS BOOTS ALLIANCE INC' | 16-Feb-96 | 17-Jul-15 | 144 | 9.77 |


| WFC | 'WELLS FARGO \& CO' | $15-$ Mar-96 | $17-J u l-15$ | 153 | 8.82 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| WMT | 'WAL-MART STORES INC' | 16 -Feb-96 | 17-Jul-15 | 155 | 8.25 |
| XOM | 'EXXON MOBIL CORP' | $15-$ Mar-96 | 17-Jul-15 | 154 | 9.21 |

Table 1: The table provides the complete list of the securities analysed together with the data coverage, the number of swap strategies considered and the average number of options per strategy.

Average skewness risk premium

|  | Before crisis <br> $(1996-2007)$ | During crisis <br> $(2007-2009)$ | After the crisis <br> $(2009-2015)$ |
| :--- | :---: | :---: | :---: |
| Average risk <br> premium | -0.0518 | -0.4281 | 1.2820 |
| Number of stocks <br> with a significant <br> risk premium | 10 | 2 | 93 |
| Average fixed leg <br> of the swap | -0.3373 | -0.5132 | -1.3579 |
| Average floating <br> leg of the swap | -0.3891 | -0.9413 | -0.0759 |

Table 2: The table shows the average risk premium across the stocks in the pre/post crisis subsamples, as well as the average fixed leg of the swap and the average floating leg. The number of significance are computed with standard t-statistics.
The individual risk premium pre/post crisis

|  | Ticker | Skewness risk premium |  |  |  |  |  | Fixed leg 1996-2007 | 2009-2 |  | Floating leg 1996-2007 2009- |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Mean | t-stat | Mean | t-stat | Mean | t-stat | Mean | Mean | t_stat | Mean | Mean | t-stat |
| 0 | SPX | 3.075 | 4.275 | 0.621 | 0.420 | 4.062 | 3.497 | -3.648 | -4.889 | -1.223 | -0.573 | -0.827 | -0.268 |
| 1 | AAPL | 0.065 | 0.093 | -0.127 | -0.272 | 0.588 | 4.517 | -0.078 | -0.620 | -5.487 | -0.013 | -0.031 | -0.026 |
| 2 | ABBV | NaN | NaN | NaN | NaN | 1.123 | 2.566 | NaN | -1.140 | NaN | NaN | -0.017 | NaN |
| 3 | ABT | 0.093 | 0.414 | 0.277 | 0.741 | 1.225 | 5.038 | -0.428 | -1.286 | -3.811 | -0.336 | -0.060 | 1.108 |
| 4 | ACN | 0.235 | 0.339 | 0.182 | 0.352 | 1.411 | 6.002 | -0.762 | -1.276 | -1.869 | -0.528 | 0.135 | 0.947 |
| 5 | AGN | 0.413 | 0.785 | -1.672 | -0.750 | 0.998 | 5.426 | -0.361 | -0.747 | -1.692 | 0.052 | 0.251 | 0.355 |
| 6 | AIG | 0.065 | 0.252 | -0.285 | -0.383 | 0.726 | 2.611 | -0.487 | -0.760 | -1.262 | -0.422 | -0.034 | 1.186 |
| 7 | ALL | 0.072 | 0.148 | -2.243 | -1.157 | 0.893 | 2.633 | -0.162 | -1.162 | -3.444 | -0.090 | -0.269 | -0.330 |
| 8 | AMGN | 0.771 | 2.361 | NaN | NaN | 1.660 | 4.725 | -0.574 | -1.444 | -2.256 | 0.197 | 0.216 | 0.046 |
| 9 | AMZN | 0.210 | 0.949 | 0.317 | 0.859 | 0.949 | 6.221 | -0.332 | -0.702 | -3.843 | -0.122 | 0.247 | 1.471 |
| 10 | AXP | -0.255 | -0.577 | -0.021 | -0.039 | 0.733 | 3.071 | -0.359 | -0.956 | -3.520 | -0.614 | -0.223 | 0.800 |
| 11 | BA | 0.718 | 0.834 | -0.645 | -1.318 | 1.556 | 5.236 | -0.491 | -1.554 | -4.058 | 0.228 | 0.003 | -0.243 |
| 12 | BAC | 0.486 | 2.316 | -0.007 | -0.012 | -0.741 | -1.183 | -0.603 | -0.231 | 3.408 | -0.118 | -0.972 | -1.319 |
| 13 | BIIB | -0.536 | -0.763 | 0.044 | 0.081 | 0.945 | 3.807 | -0.307 | -0.787 | -2.734 | -0.843 | 0.158 | 1.362 |
| 14 | BK | 0.503 | 1.239 | 0.540 | 1.424 | 1.681 | 5.400 | -0.280 | -1.684 | -4.449 | 0.223 | -0.003 | -0.540 |
| 15 | BLK | 0.032 | 0.092 | -0.234 | -0.421 | 0.417 | 1.135 | -0.077 | -0.861 | -3.342 | -0.044 | -0.445 | -1.006 |
| 16 | BMY | 0.136 | 0.550 | 0.174 | 0.618 | 1.300 | 4.166 | -0.279 | -1.197 | -3.305 | -0.143 | 0.103 | 0.847 |
| 17 | BRK | NaN | NaN | NaN | NaN | 1.585 | 4.496 | NaN | -1.692 | NaN | NaN | -0.107 | NaN |
| 18 | C | 0.340 | 1.383 | 0.478 | 0.574 | 1.867 | 4.050 | -0.485 | -1.839 | -3.023 | -0.145 | 0.029 | 0.608 |
| 19 | CAT | 0.678 | 1.510 | 0.205 | 0.623 | 0.788 | 3.930 | -0.342 | -0.834 | -2.750 | 0.336 | -0.046 | -0.805 |
| 20 | CELG | -0.155 | -0.437 | -0.208 | -0.438 | 1.169 | 4.700 | -0.182 | -1.251 | -5.253 | -0.337 | -0.082 | 0.664 |
| 21 | CL | 0.895 | 1.579 | 1.397 | 0.765 | 1.586 | 5.167 | -0.190 | -1.547 | -4.232 | 0.705 | 0.040 | -1.158 |
| 22 | CMCSA | 0.140 | 0.201 | -0.288 | -0.617 | 0.766 | 3.446 | -0.299 | -0.780 | -2.706 | -0.159 | -0.013 | 0.208 |
| 23 | COF | -0.694 | -1.187 | -0.084 | -0.166 | 1.812 | 5.671 | -0.482 | -1.714 | -3.976 | -1.176 | 0.098 | 2.158 |
| 24 | COP | -1.012 | -1.023 | -1.591 | -0.775 | 1.272 | 2.605 | -0.513 | -1.520 | -2.886 | -1.525 | -0.249 | 1.220 |
| 25 | cost | 0.111 | 0.381 | 0.658 | 1.146 | 2.194 | 5.745 | -0.433 | -1.957 | -3.994 | -0.323 | 0.237 | 1.662 |
| 26 | CSCO | 0.061 | 0.363 | -0.733 | -1.106 | 0.386 | 2.234 | -0.422 | -0.551 | -1.362 | -0.360 | -0.165 | 0.855 |



$\stackrel{\infty}{\stackrel{\infty}{C}} \underset{\sim}{\circ}$ Z
-0.117
-0.148
0.288
-0.092
0.230
0.058
0.000
-0.458

| -6.011 | 0.501 |
| :--- | :--- |
| -4.588 | -0.597 |
| NaN | NaN |
| -4.967 | 0.672 |
| -3.733 | -6.232 |
| -1.986 | -0.969 |
| -3.732 | -0.598 |
| -4.549 | -0.060 |


|  |
| :---: |
|  |






Table 3: The table reports in the first six columns the average individual skewness risk premiums in the three subsamples: before, during and after the financial crisis of 2007, together with the t-statistics. The columns below the headline 'Fixed leg' report the individual average fixed leg of the skew swap before and after the crisis together with the t-statistic of the difference. The columns below the headline 'Floating leg' report the individual average floating leg of the skew
swap before and after the crisis together with the t-statistic of the difference.

## Moneyness categories

| Category | Delta call options | Delta put options |
| :---: | :---: | :---: |
| 1 | $0.875<\Delta_{C} \leq 0.98$ | $-0.125<\Delta_{p} \leq-0.02$ |
| 2 | $0.625<\Delta_{C} \leq 0.875$ | $-0.375<\Delta_{p} \leq-0.125$ |
| 3 | $0.375<\Delta_{C} \leq 0.625$ | $-0.625<\Delta_{p} \leq-0.375$ |
| 4 | $0.125<\Delta_{C} \leq 0.375$ | $-0.875<\Delta_{p} \leq-0.625$ |
| 5 | $0.02<\Delta_{C} \leq 0.125$ | $-0.98<\Delta_{p} \leq-0.875$ |

Table 4: The table shows the five categories of moneyness in which the options are classified according to their deltas. The category 1 contains the most out-of-the-money puts and in-themoney calls, while the category 5 contains the most out-of-the-money calls and in-the-money puts.

## Bakshi, Kapadia, Madan (2003) ex-ante skewness

| Q-skewness |  |  |  | Q-skewness |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1996-2007 2009-20 |  |  |  | 1996-2007 2009 |  |  |  |
| Ticker | Mean | Mean | t _stat | Ticker | Mean | Mean | t-stat |
| AAPL | -0.030 | -0.988 | -5.670 | INTC | -0.325 | -1.754 | -2.364 |
| ABBV | NaN | -2.005 | NaN | JNJ | -1.415 | -4.139 | -3.288 |
| ABT | -0.714 | -2.146 | -3.322 | JPM | -0.796 | -3.983 | -3.186 |
| ACN | -1.292 | -2.152 | -1.784 | KMI | NaN | -0.254 | NaN |
| AGN | -0.531 | -1.419 | -2.023 | KO | -0.590 | -4.015 | -2.894 |
| AIG | -0.693 | -1.629 | -2.091 | LLY | -0.779 | -3.300 | -3.830 |
| ALL | -0.257 | -2.472 | -3.472 | LMT | -0.178 | -2.652 | -4.173 |
| AMGN | -0.807 | -2.546 | -2.317 | LOW | -0.529 | -2.376 | -1.331 |
| AMZN | -0.408 | -1.146 | -4.083 | MA | -0.156 | -1.283 | -4.014 |
| AXP | -0.489 | -1.597 | -3.237 | MCD | -0.382 | -3.147 | -3.125 |
| BA | -0.730 | -2.685 | -3.914 | MDLZ | -0.237 | -2.299 | -4.201 |
| BAC | -0.935 | -0.283 | 3.350 | MDT | -0.957 | -2.403 | -2.894 |
| BIIB | -0.423 | -1.438 | -2.986 | MET | -0.603 | -4.140 | -4.591 |
| BK | -0.394 | -3.611 | -3.083 | MMM | -0.536 | -4.392 | -4.573 |
| BLK | -0.050 | -1.394 | -3.498 | MO | -0.527 | -1.799 | -3.955 |
| BMY | -0.385 | -2.322 | -2.950 | MON | 0.273 | -2.434 | -4.909 |
| BRK | NaN | -3.752 | NaN | MRK | -0.797 | -4.909 | -4.920 |
| C | -0.708 | -4.485 | -2.989 | MS | -0.286 | -1.649 | -3.717 |
| CAT | -0.493 | -1.384 | -2.634 | MSFT | -0.594 | -2.968 | -2.455 |
| CELG | -0.181 | -2.719 | -3.297 | NEE | 0.420 | -2.838 | -4.670 |
| CL | -0.240 | -2.762 | -3.837 | NKE | -0.920 | -3.071 | -3.175 |
| CMCSA | -0.581 | -1.213 | -2.119 | ORCL | -0.422 | -2.511 | -3.825 |
| COF | -0.664 | -3.767 | -3.859 | OXY | -0.236 | -1.583 | -2.832 |
| COP | -0.775 | -74.592 | -1.023 | PCLN | 0.075 | -0.671 | -4.136 |
| COST | -0.639 | -3.663 | -3.657 | PEP | -0.822 | -2.622 | -2.756 |
| CSCO | -0.606 | -0.821 | -1.374 | PFE | -0.385 | -1.812 | -3.108 |
| CVS | -0.676 | -4.267 | -2.995 | PG | -0.950 | -2.898 | -2.465 |
| CVX | -0.366 | -4.782 | -5.372 | PM | NaN | -3.390 | NaN |
| DD | -0.460 | -3.628 | -3.687 | PYPL | NaN | -2.746 | NaN |
| DHR | -0.691 | -1.763 | -2.615 | QCOM | -0.399 | -1.689 | -2.363 |


| DIS | -0.336 | -3.359 | -3.578 | RTN | -0.185 | -2.507 | -3.726 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| DOW | -0.507 | -2.182 | -3.220 | SBUX | -0.312 | -5.234 | -1.669 |
| DUK | -0.094 | -2.417 | -4.203 | SLB | -0.207 | -4.124 | -2.439 |
| EMC | -0.423 | -1.364 | -2.953 | SO | -1.058 | -1.466 | -0.565 |
| EMR | -0.057 | -3.761 | -4.357 | SPG | -0.542 | -3.066 | -3.868 |
| EXC | -0.523 | -3.620 | -3.083 | T | -0.474 | -3.193 | -4.000 |
| F | -0.523 | -0.517 | 0.031 | TGT | -1.074 | -93.017 | -1.014 |
| FB | NaN | -0.507 | NaN | TWX | -0.348 | -4.438 | -5.618 |
| FDX | -0.245 | -3.050 | -4.356 | TXN | -0.313 | -1.973 | -4.337 |
| FOXA | 1.038 | -1.809 | -2.507 | UNH | -0.505 | -1.668 | -2.779 |
| GD | -0.640 | -2.500 | -3.629 | UNP | -0.677 | -2.984 | -3.520 |
| GE | -0.719 | -2.035 | -2.771 | UPS | -0.728 | -3.695 | -4.216 |
| GILD | -0.492 | -1.997 | -4.172 | USB | 0.139 | -3.378 | -4.935 |
| GM | NaN | -1.635 | NaN | UTX | -0.864 | -3.438 | -4.439 |
| GOOGL | -0.219 | -1.021 | -4.840 | V | NaN | -3.579 | NaN |
| GS | -0.600 | -2.486 | -3.702 | VZ | -0.547 | -3.940 | -4.560 |
| HAL | -0.435 | -3.130 | -4.160 | WBA | -0.538 | -409.599 | -1.006 |
| HD | -1.121 | -5.683 | -1.795 | WFC | -1.310 | -3.197 | -2.148 |
| HON | -0.457 | -4.353 | -2.654 | WMT | -0.647 | -2.963 | -3.156 |
| IBM | -0.893 | -5.865 | -4.004 | XOM | -0.648 | -3.876 | -4.418 |

Table 5: The table reports the individual average $\mathbb{Q}$ skewness before and after the crisis together with the t -statistic of the difference. The time series of the $\mathbb{Q}$ skewnesses are calculated monthly with the metodology of Bakshi et al. (2003).

## Predictive regression

|  | $\overline{\alpha_{0}}$ | $N_{\alpha_{0}}$ | $\overline{\alpha_{1}}$ | $N_{\alpha_{1}}$ | $R^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| full sample | -0.7367 | 17 | 0.1205 | 5 | 0.0082 |
| pre crisis | -0.5339 | 7 | 0.5016 | 24 | 0.0351 |
| post crisis | -0.3076 | 4 | -0.0287 | 1 | 0.0085 |

Table 6: The table shows the average results of the time series regressions $f l l_{i, t}=\alpha_{0}+\alpha_{1} f x l_{i, t}+$ $\epsilon_{t}$, where $f l l_{i, t}$ is the the floating leg ( $\mathbb{P}$ skewness) of the skew swap of the month $t$ for the stock $i$ and $f x l_{i, t}$ is the fixed leg ( $\mathbb{Q}$ skewness) of the same skew swap. $\overline{\alpha_{0}}$ is the average estimate of $\alpha_{0}$ and $N_{\alpha_{0}}$ is the number of stocks for which $\alpha_{0}$ is significant. $\overline{\alpha_{1}}$ is the average estimate of $\alpha_{1}$ and $N_{\alpha_{1}}$ is the number of stocks for which $\alpha_{1}$ is significant.


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